On Fuzzy Ideals and Level Subsets of Ordered Γ-Groupoids

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Abstract. We characterize the fuzzy left (resp. right) ideals, the fuzzy ideals and the fuzzy prime (resp. semiprime) ideals of an ordered Γ-groupoid M in terms of level subsets and we prove that the cartesian product of two fuzzy left (resp. right) ideals of M is a fuzzy left (resp. right) ideal of $M \times M$, and the cartesian product of two fuzzy prime (resp. semiprime) ideals of M is a fuzzy prime (resp. semiprime) ideal of $M \times M$. As a result, if μ and σ are fuzzy left (resp. right) ideals, ideals, fuzzy prime or fuzzy semiprime ideals of M, then the nonempty level subsets $(\mu \times \sigma)_t$ are so.

1 Introduction and prerequisites

For two nonempty sets M and Γ, we denote by the letters of the English alphabet the elements of M and by the letters of the Greek alphabet the elements of Γ, and define $M\Gamma M := \{a\gamma b \mid a, b \in M, \gamma \in \Gamma\}$. Then M is called a Γ-*groupoid* if

(1) $M\Gamma M \subseteq M$ and

(2) if $a, b, c, d \in M$ and $\gamma, \mu \in \Gamma$ such that $a = c$, $b = d$ and $\gamma = \mu$, then
  have $a\gamma b = c\mu d$.

If, in addition, for all $a, b \in M$ and all $\gamma, \mu \in \Gamma$, $a\gamma(b\mu c) = (a\gamma b)\mu c$, then M is called a Γ-*semigroup*. For $\Gamma = \{\gamma\}$, the Γ-groupoid M is the so called groupoid and the ordered Γ-groupoid the ordered groupoid (: *po*-groupoid), the Γ-semigroup is the semigroup and the ordered Γ-semigroup the ordered semigroup (where γ is the multiplication on M usually denoted by "·"). An *ordered* Γ-*groupoid* (: *po*-Γ-*groupoid*) is a Γ-groupoid M with an order relation "$\leq$" on M such that $a \leq b$ implies $a\gamma c \leq b\gamma c$ and $c\gamma a \leq c\gamma b$ for all $\gamma \in \Gamma$. Let M be a *po*-Γ-groupoid. A subset T of M is called *prime* if for every $a, b \in M$ and every $\gamma \in \Gamma$ such that $a\gamma b \in T$, we have $a \in T$ or $b \in T$. A subset T of M is called *semiprime* if for every $a \in M$ and every $\gamma \in \Gamma$ such that $a\gamma a \in T$, we have $a \in T$. A nonempty subset A of M is called a *left* (resp. *right*) *ideal* of M if

(1) $M\Gamma A \subseteq A$ (resp. $A\Gamma M \subseteq A$) and

(2) if $a \in A$ and $M \ni b \le a$ implies $b \in A$.

If the set A is both a left and a right ideal of M, then it is called an *ideal* of M. If M is an ordered Γ-groupoid, then any mapping $\mu : M \to [0,1]$ is called a *fuzzy subset* of M (or a *fuzzy set* in M) (L. Zadeh). The mapping μ is called a *fuzzy left ideal* of M if

(1) $\mu(x\gamma y) \ge \mu(y)$ for every $x, y \in M$ and every $\gamma \in \Gamma$ and

(2) if $x \le y$ implies $\mu(x) \ge \mu(y)$.

It is called a *fuzzy right ideal* of M if

(1) $\mu(x\gamma y) \ge \mu(x)$ for every $x, y \in M$ and every $\gamma \in \Gamma$ and

(2) if $x \le y$ implies $\mu(x) \ge \mu(y)$.

A fuzzy subset which is both a fuzzy left and a fuzzy right ideal of M is called a *fuzzy ideal* of M. A fuzzy subset μ of M is a fuzzy ideal of M if and only if

(1) $\mu(x\gamma y) \ge \max\{\mu(x), \mu(y)\}$ for every $x, y \in M$ and every $\gamma \in \Gamma$ and

(2) if $x \le y$ implies $\mu(x) \ge \mu(y)$.

A characterization of fuzzy prime subsets of a semigroup in terms of level subsets has been considered in [4; Lemma 2.3] in which λ should be replaced by t (or t should be replaced by λ) and the commutativity of the semigroup is not necessary as the same holds in semigroups is general. As an immediate consequence of the Lemma 2.3 in [4], a fuzzy ideal of a semigroup S is prime if and only if for every $t \in [0,1]$ the t-level subset $f_t := \{x \in S \mid f(x) \ge t\}$ (of S), if it is nonempty, is a prime ideal of S. This is the Theorem 3.1 in [3] in which the "$xy \subseteq \mu_t$" (in the second line of the proof) should be replaced by "$xy \in \mu_t$" and the proof of the converse statement (lines 5–11 of the proof) should be corrected. A characterization of fuzzy semiprime ideals of a semigroup in terms of level subsets has been considered in the Theorem 3.2 in [3] but the proof of the "converse" statement in it should be corrected. The cartesian product of two fuzzy left (resp. fuzzy right) ideals of semigroups and the cartesian product of two fuzzy prime (resp. fuzzy semiprime) ideals of a semigroup has been studied in [3]. On the other hand, a characterization of fuzzy prime and fuzzy semiprime ideals of ordered semigroups in terms of level subsets has been considered in the Theorems 2.6 and 2.7 in [6] from which the Theorems 3.1 and 3.2 in [3] are also obtained. For a characterization of fuzzy ideals of ordered semigroup in terms of level subsets see the Lemma 2.4 in [6] and the Lemma 2.7 in [5]. The reference in Lemma 2.4 in [6] should be corrected. In the "converse statement" of the proof of Theorem 3.1 in [3] as well as in the proof of Theorem 3.2 in [3], the phrase "Let every nonempty subset μ_t of μ be a prime (semiprime) ideal of S" is better to be replaced by the phrase "Let every

subset μ_t of μ be a prime (semiprime) ideal of S", and this is because the ideals are, by definition, nonempty sets. Finally, the proofs of Propositions 4.4 and 4.6 in [3] should be omitted as they are immediate consequences of Propositions 4.2 and 4.3 and the Theorem 3.2 given in the same paper.

In the present paper we first characterize the fuzzy left (right) ideals, the fuzzy ideals, the fuzzy prime and the fuzzy semiprime ideals of an ordered Γ-groupoid in terms of level subsets. Then we prove that the cartesian product of two fuzzy left (resp. fuzzy right) ideals of an ordered Γ-groupoid M is a fuzzy left (resp. fuzzy right) ideal of $M \times M$. Thus the cartesian product of two fuzzy ideals of M is a fuzzy ideal of $M \times M$. Moreover, the cartesian product of two fuzzy prime (resp. fuzzy semiprime) ideals of a Γ-groupoid M is a fuzzy prime (resp. fuzzy semiprime) ideal of $M \times M$. As a consequence, if μ and σ are fuzzy left (resp. fuzzy right) ideals of an ordered Γ-groupoid M then, for any $t \in [0,1]$ if the level subset $(\mu \times \sigma)_t$ is nonempty, then it is a left (resp. right) ideal of $M \times M$. If μ and σ are fuzzy ideals of M and the level subset $(\mu \times \sigma)_t$ is nonempty, then it is an ideal of $M \times M$. If μ and σ are fuzzy prime (resp. fuzzy semiprime) ideals of M, then the nonempty level subsets $(\mu \times \sigma)_t$ are prime (resp. semiprime) ideals of $M \times M$. The present paper serves as an example to show the way we pass from fuzzy ordered groupoids (resp. fuzzy ordered semigroups) to fuzzy ordered Γ-groupoids (resp. fuzzy ordered Γ-semigroups) and from fuzzy groupoids (resp. fuzzy semigroups) to fuzzy Γ-groupoids (resp. fuzzy Γ-semigroups). On the other hand, from the results of Γ-groupoids or Γ-ordered groupoids where $\Gamma = \{\gamma\}$ (γ being a symbol) the corresponding results on groupoids or ordered groupoids are obtained. The fuzzy sets in ordered groupoids have been introduced in [1] and one can find several papers on fuzzy ordered semigroups in the bibliography.

2 Characterization of prime and semiprime fuzzy ideals in terms of level subsets

Following the terminology of fuzzy prime subset of a groupoid introduced in [1, 2], we give the following definition

Definition 1. Let M be an ordered Γ-groupoid (or a Γ-groupoid). A fuzzy subset μ of M is called *fuzzy prime subset of M* or *prime fuzzy subset of M* if

$$\mu(x\gamma y) \leq \max\{\mu(x), \mu(y)\}$$

for all $x, y \in M$ and all $\gamma \in \Gamma$.

Recall that if μ is a fuzzy prime ideal of M, then for every $x, y \in M$ and every $\gamma \in \Gamma$, we have $\mu(x\gamma y) = \max\{\mu(x), \mu(y)\}$. So a fuzzy ideal μ of M can be called *prime* if $\mu(x\gamma xy) = \max\{\mu(x), \mu(y)\}$ for all $x, y \in M$ and all $\gamma \in \Gamma$.

Definition 2. Let M be an ordered Γ-groupoid (or a Γ-groupoid). A fuzzy subset μ of M is called *fuzzy semiprime subset of M* or *semiprime fuzzy subset of M* if

$$\mu(x) \geq \mu(x\gamma x)$$

for every $x \in M$ and every $\gamma \in \Gamma$.

Notation 3. If μ is a fuzzy subset of an ordered Γ-groupoid (or a Γ-groupoid) M then, for any $t \in [0, 1]$ (: the closed interval of real numbers), we denote by μ_t the subset of M defined by

$$\mu_t := \{x \in M \mid \mu(x) \geq t\}.$$

The set μ_t is called the *t-level subset* or just *level subset* of μ.

Theorem 4. *Let M be an ordered Γ-groupoid. If μ is a fuzzy left ideal of M and $\mu_t \neq \emptyset$, then μ_t is a left ideal of M. "Conversely", if μ_t is a left ideal of M for every t, then μ is a fuzzy left ideal of M.*

Proof. $\Longrightarrow$. Suppose μ is a fuzzy left ideal of M and $\mu_t \neq \emptyset$ for some $t \in [0, 1]$. Then $M\Gamma\mu_t \subseteq \mu_t$. Indeed: Let $a \in M$, $\gamma \in \Gamma$ and $b \in \mu_t$. Since μ is a fuzzy left ideal of M, we have $\mu(a\gamma b) \geq \mu(b)$. Since $b \in \mu_t$, we have $\mu(b) \geq t$. Then $\mu(a\gamma b \geq t$, and $a\gamma b \in \mu_t$. Let $a \in \mu_t$ and $M \ni b \leq a$. Then $b \in \mu_t$. Indeed: Since $a \in \mu_t$, we have $\mu(a) \geq t$. Since $b \leq a$ and μ is a fuzzy left ideal of M, we have $\mu(b) \geq \mu(a)$. Then $\mu(b) \geq t$, and $b \in \mu_t$. Thus μ_t is a left ideal of M.

$\Longleftarrow$. Suppose μ_t is a left ideal of M for every t and let $a, b \in M$ and $\gamma \in \Gamma$. Then $\mu(a\gamma b) \geq \mu(b)$. Indeed: Since $\mu(b) \in [0, 1]$ and $\mu(b) \geq \mu(b)$, we have $b \in \mu_{\mu(b)}$. Since $\mu_{\mu(b)}$ is a left ideal of M, we have $a\gamma b \in M\Gamma\mu_{\mu(b)} \subseteq \mu_{\mu(b)}$. Then $a\gamma b \in \mu_{\mu(b)}$, and $\mu(a\gamma b) \geq \mu(b)$. Let now $a \leq b$. Then $\mu(a) \geq \mu(b)$. Indeed: Since $b \in \mu_{\mu(b)}$, $M \ni a \leq b$ and $\mu_{\mu(b)}$ is a left ideal of M, we have $a \in \mu_{\mu(b)}$, then $\mu(a) \geq \mu(b)$. $\qquad \Box$

In a similar way we have the following

Theorem 5. *Let M be an ordered Γ-groupoid. If μ is a fuzzy right ideal of M and $\mu_t \neq \emptyset$, then μ_t is a right ideal of M. "Conversely", if μ_t is a right ideal of M for every t, then μ is a fuzzy right ideal of M.*

By Theorems 4 and 5, we have the following theorem

Theorem 6. *Let M be an ordered Γ-groupoid. If μ is a fuzzy ideal of M and $\mu_t \neq \emptyset$ for some $t \in [0, 1]$, then μ_t is an ideal of M. "Conversely", if μ_t is an ideal of M for every $t \in [0, 1]$, then μ is a fuzzy ideal of M.*

Lemma 7. *Let M be an ordered Γ-groupoid. Then μ is a fuzzy prime subset of M if and only if the level subset μ_t is a prime subset of M for every t.*

Proof. $\Longrightarrow$. Let $a, b \in M$ and $\gamma \in \Gamma$ such that $a\gamma b \in \mu_t$. Then $a \in \mu_t$ or $b \in \mu_t$. Indeed: Since $a\gamma b \in \mu_t$, we have $\mu(a\gamma b) \geq t$. Since μ is a fuzzy prime subset of M, we have $\mu(a\gamma b) \leq \max\{\mu(a), \mu(b)\}$. Since $\mu(a), \mu(b) \in [0, 1]$, we have $\mu(a) \leq \mu(b)$ or $\mu(b) \leq \mu(a)$. If $\mu(a) \leq \mu(b)$, then $\max\{\mu(a), \mu(b)\} = \mu(b)$, and $t \leq \mu(b)$, so $b \in \mu_t$. If $\mu(b) \leq \mu(a)$, then $t \leq \mu(a\gamma b) = \mu(a)$, and $a \in \mu_t$.

$\Longleftarrow$. Suppose μ_t is a prime subset of M for every t and let $x, y \in M$ and $\gamma \in \Gamma$. Then $\mu(x\gamma y) = \max\{\mu(x), \mu(y)\}$. Indeed: Since $x\gamma y \in \mu_{\mu(x\gamma y)}$, by hypothesis, we have $x \in \mu_{\mu(x\gamma y)}$ or $y \in \mu_{\mu(x\gamma y)}$. Then $\mu(x) \geq \mu(x\gamma y)$ or $\mu(y) \geq \mu(x\gamma y)$, thus $\max\{\mu(x), \mu(y)\} \geq \mu(x\gamma y)$. $\square$

By Theorem 6 and Lemma 7, we have the following theorem

Theorem 8. *Let M be an ordered Γ-groupoid. If μ is a fuzzy prime ideal of M and $\mu_t \neq \emptyset$, then μ_t is a prime ideal of M. "Conversely", if μ_t is a prime ideal of M for every t, then μ is a fuzzy prime ideal of M.*

Lemma 9. *Let M be an ordered Γ-groupoid. Then μ is a fuzzy semiprime subset of M if and only if the level subset μ_t is a semiprime subset of M for every t.*

Proof. $\Longrightarrow$. Let $t \in [0, 1]$, $a \in M$ and $\gamma \in \Gamma$ such that $a\gamma a \in \mu_t$. Then $a \in \mu_t$. Indeed: Since μ is a fuzzy semiprime subset of M, we have $\mu(a) \geq \mu(a\gamma a)$. Since $a\gamma a \in \mu_t$, we have $\mu(a\gamma a) \geq t$. Then $\mu(a) \geq t$, and $a \in \mu_t$.

$\Longleftarrow$. Let $a \in M$ and $\gamma \in \Gamma$. Then $\mu(a) \geq \mu(a\gamma a)$. Indeed: By hypothesis, $\mu_{\mu(a\gamma a)}$ is a semiprime subset of M. Since $a\gamma a \in \mu_{\mu(a\gamma a)}$, we have $a \in \mu_{\mu(a\gamma a)}$, then $\mu(a) \geq \mu(a\gamma a)$, so μ is fuzzy semiprime. $\square$

By Theorem 6 and Lemma 9, we have the following theorem

Theorem 10. *Let M be an ordered Γ-groupoid. If μ is a fuzzy semiprime ideal of M and $\mu_t \neq \emptyset$, then μ_t is a semiprime ideal of M. "Conversely", if μ_t is a semiprime ideal of M for every t, then μ is a fuzzy semiprime ideal of M.*

As a consequence, given a groupoid or an ordered groupoid G, if μ is a fuzzy left ideal, fuzzy right ideal, fuzzy ideal, fuzzy prime ideal or fuzzy semiprime ideal of G, respectively, then the nonempty level subsets μ_t of μ are left ideals, right ideals, ideals,

prime ideals or semiprime ideals of G, respectively. "Conversely" if for a fuzzy subset μ of G the and any $t \in [0,1]$ the level subset μ_t of μ is a left ideal, right ideal, ideal, prime ideal or semiprime ideal of G, respectively, then μ is a fuzzy left ideal, fuzzy right ideal, fuzzy ideal, fuzzy prime ideal or fuzzy semiprime ideal of G, respectively.

3   Cartesian product of fuzzy ideals, fuzzy prime and fuzzy semiprime ideals

If $(M, \leq, \Gamma)$ is an ordered Γ-groupoid, $M \times M := \{(x,y) \mid x, y \in M\}$ and for any $(a,b), (c,d) \in M \times M$ and any $\gamma \in \Gamma$ we define

$$(a,b)\gamma(c,d) := (a\gamma c, b\gamma d),$$

then $(M \times M, \leq, \Gamma)$ is an ordered Γ-groupoid as well.

For two fuzzy subsets μ and σ of an ordered Γ-groupoid M, the cartesian product of μ and σ is the fuzzy subset of $M \times M$ defined by

$$\mu \times \sigma : M \times M \to [0,1] \mid (x,y) \to \min\{\mu(x), \sigma(y)\}.$$

That is,

$$(\mu \times \sigma)\Big((x,y)\Big) := \min\{\mu(x), \sigma(y)\}$$

for every $x, y \in M$. As no confusion is possible, we write $(\mu \times \sigma)(x,y)$ instead of $(\mu \times \sigma)\Big((x,y)\Big)$.

Theorem 11. *Let M be an ordered Γ-groupoid and μ, σ fuzzy left (resp. right) ideals of M. Then $\mu \times \sigma$ is a fuzzy left (resp. right) ideal of $M \times M$.*

Proof. Suppose μ and σ are fuzzy left ideals of M. Let $(a,b), (c,d) \in M \times M$ and $\gamma \in \Gamma$. Then

$$(\mu \times \sigma)\Big((a,b)\gamma(c,d)\Big) \geq (\mu \times \sigma)(c,d).$$

Indeed:

$$
\begin{aligned}
(\mu \times \sigma)\Big((a,b)\gamma(c,d)\Big) &= (\mu \times \sigma)(a\gamma c, b\gamma d) = \min\{\mu(a\gamma c), \sigma(b\gamma d)\} \\
&\geq \min\{\mu(c), \sigma(d)\} \text{ (since } \mu, \sigma \text{ are fuzzy left ideals of } M) \\
&= (\mu \times \sigma)(c,d).
\end{aligned}
$$

Let now $(a,b) \leq (c,d)$. Then

$$
\begin{aligned}
(\mu \times \sigma)(a,b) &= \min\{\mu(a), \mu(b)\} \\
&\leq \min\{\mu(c), \mu(d)\} \text{ (since } a \leq c, b \leq d) \\
&= (\mu \times \sigma)(c,d).
\end{aligned}
$$

Thus $\mu \times \sigma$ is a fuzzy left ideal of M. Similarly the cartesian product of two fuzzy right ideals of M is a fuzzy right ideal of $M \times M$. $\quad\square$

By Theorem 11, the following theorem holds

Theorem 12. *Let M be an ordered Γ-groupoid and μ, σ fuzzy ideals of M. Then $\mu \times \sigma$ is a fuzzy ideal of $M \times M$.*

Lemma 13. *Let M be an ordered Γ-groupoid and μ, σ fuzzy prime subsets of M. Then $\mu \times \sigma$ is a fuzzy prime subset of $M \times M$.*

Proof. Let $(a,b),(c,d) \in M \times M$ and $\gamma \in \Gamma$. Then

$$(\mu \times \sigma)\Big((a,b)\gamma(c,d)\Big) = \max\{(\mu \times \sigma)(a,b),(\mu \times \sigma)(c,d)\}.$$

In fact:

$$
\begin{aligned}
(\mu \times \sigma)\Big((a,b)\gamma(c,d)\Big) &= (\mu \times \sigma)(a\gamma c, b\gamma d) = \min\{\mu(a\gamma c), \sigma(b\gamma d)\} \\
&= \min\Big\{\max\{\mu(a),\mu(c)\}, \max\{\sigma(b),\sigma(d)\}\Big\} \\
&= \max\Big\{\min\{\mu(a),\sigma(b)\}, \min\{\mu(c),\sigma(d)\}\Big\} \\
&= \max\{(\mu \times \sigma)(a,b),(\mu \times \sigma)(c,d)\}.
\end{aligned}
$$

$\quad\square$

By Theorem 12 and Lemma 13, we have the following theorem

Theorem 14. *If M is an ordered Γ-groupoid and μ, σ fuzzy prime ideals of M, then $\mu \times \sigma$ is a fuzzy prime ideal of $M \times M$.*

Lemma 15. *Let M be an ordered Γ-groupoid and μ, σ fuzzy semiprime subsets of M. Then $\mu \times \sigma$ is a fuzzy semiprime subset of $M \times M$.*

Proof. Let $(a,b) \in M$ and $\gamma \in \Gamma$. Then

$$(\mu \times \sigma)(a,b) \geq (\mu \times \sigma)\Big((a,b)\gamma(a,b)\Big).$$

Indeed: $(\mu \times \sigma)(a,b) = \min\{\mu(a),\sigma(b)\}$. Since μ and σ are fuzzy semiprime subsets of M, we have $\mu(a) \geq \mu(a\gamma a)$ and $\sigma(b) \geq \sigma(b\gamma b)$. Then we get

$$
\begin{aligned}
(\mu \times \sigma)(a,b) &\geq \min\{\mu(a\gamma a), \sigma(b\gamma b)\} = (\mu \times \sigma)(a\gamma a, b\gamma b) \\
&= (\mu \times \sigma)\Big((a,b)\gamma(a,b)\Big).
\end{aligned}
$$

$\quad\square$

By Theorem 12 and Lemma 15, we have the following theorem

Theorem 16. *If M is an ordered Γ-groupoid and μ, σ fuzzy semiprime ideals of M, then $\mu \times \sigma$ is a fuzzy semiprime ideal of $M \times M$.*

By Theorems 4–6 and 11, 12, we have the following corollary

Corollary 17. *If M is an ordered Γ-groupoid and μ, σ fuzzy left (resp. right) ideals of M, then the level subset $(\mu \times \sigma)_t$, if it is nonempty, is a left (resp. right) ideal of $M \times M$. If μ and σ are fuzzy ideals of M and the level subset $(\mu \times \sigma)_t$ is nonempty, then it is an ideal of $M \times M$.*

By Theorems 8, 10, 14 and 16, we have the following

Corollary 18. *If M is an ordered Γ-groupoid and μ, σ fuzzy prime (resp. semiprime) ideals of M, then the level subset $(\mu \times \sigma)_t$, if it is nonempty, is a prime (resp. semiprime) ideal of $M \times M$.*

As a conclusion, if G is a groupoid or an ordered groupoid and μ, σ fuzzy left ideals, fuzzy right ideals, fuzzy ideals, fuzzy prime ideals or fuzzy semiprime ideals of G, respectively, then the cartesian product $\mu \times \sigma$ of μ and σ is, respectively so. If G is a groupoid or an ordered groupoid and μ, σ fuzzy left (right) ideals, fuzzy ideals, fuzzy prime (semiprime) ideals of G, respectively, then for any $t \in [0,1]$ for which $(\mu \times \sigma)_t$ is nonempty, the level subset $(\mu \times \sigma)_t$ is, respectively, a left (right) ideal, ideal, prime (semiprime) ideal of the cartesian product $M \times M$ of M.

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